Extracting micro-Doppler radar signatures from rotating targets using Fourier-Bessel Transform and Time-Frequency analysis

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Abstract

In this paper, we report the efficiency of Fourier Bessel transform and time-frequency based method in conjunction with the fractional Fourier transform, for extracting micro-Doppler radar signatures from the rotating targets. This approach comprises mainly of two processes; the first being decomposition of the radar return, in order to extract micro-Doppler (m-D) features and the second being, the time-frequency analysis to estimate motion parameters of the target. In order to extract m-D features from the radar signal returns, the time domain radar signal is decomposed into stationary and non-stationary components using Fourier Bessel transform in conjunction with the fractional Fourier transform. The components are then reconstructed by applying the inverse Fourier Bessel transform. After the extraction of the m-D features from the target's original radar return, time-frequency analysis is used to estimate the target's motion parameters. This proposed method is also an effective tool for detecting manoeuvring air targets in strong sea-clutter and is also applied to both simulated data and real world experimental data.

Index Terms

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Time-Frequency analysis, Short-Time Fourier Transform, Wigner Ville Distribution, Fourier Bessel Transform, Fractional Fourier Transform.

I. Introduction

Most widely used time-frequency transforms are short-time Fourier Transform (STFT) and Wigner Ville distribution (WVD). In STFT, time and frequency resolutions are limited by the size of window function used in calculating STFT. For mono-component signals, WVD gives the best time and frequency resolutions without any cross terms. However, in the case of multi component signals, the occurrence of cross terms degrade the readability of the time-frequency representation and limits the usefulness of WVD.

In order to achieve cross-term free WVD, Pachori et al. in [1] and [2] used Fourier-Bessel transform to decompose the multi-component signal, and then applied WVD to each component separately to analyze its time-frequency distribution. An improvement of this method has been proposed in [3]. This approach is applied to the multi-component signal whose signal components overlap only in the time domain. This is applied to the simulated data. But if the components of a multi-component signal overlap in both time and frequency domains then it is not possible to separate the signal components using the method in [1], [2] and [3]. However, in real-time applications, several scenarios are related to a multi-component signal whose signal components overlap in both time and frequency domains. Therefore, FBT and WVD alone can not be used in real-time applications. This paper presents a new approach, which is based on Fourier Bessel transform in conjunction with the Fractional Fourier transform (FrFT) to decompose the non-stationary signal whose component frequencies overlap in both time and frequency domains. The WVD is then applied to each component separately to analyze its time-frequency distribution. This approach is an advancement to the method used by [1] and [2] and has now several real-time applications. We have successfully demonstrated the proposed approach to experimental data sets.

This paper is basically organized into six sections. In section II, a brief introduction to time-frequency analysis, particularly STFT and WVD is presented. Sections III and IV deal with the mathematical formulation of FBT and Fractional Fourier transform (FrFT). In section V, the application of Fourier-Bessel and Time-Frequency (FB-TF) method, in removing the interference terms that occur when a multi-component signal is analyzed using WVD, is presented. Section

VI demonstrates the effectiveness of the proposed method in extracting m-D features of the rotating targets and also in the reduction of sea clutter, thus by enhancing the target detection.

II. Time-Frequency analysis

Time-Frequency techniques are broadly classified into two categories: Linear transforms and Quadratic (or bilinear) transforms.

A. Linear Time-Frequency Transforms

All those time-frequency representations that obey the principle of superposition can be classified under the linear time-frequency transforms. Some of the linear time-frequency transforms are STFT, continuous wavelet transform (CWT) and the adaptive time-frequency transforms. STFT is the most widely used time-frequency technique among the linear time-frequency transformations.

Basic principle behind STFT is segmenting the signal into narrow time intervals using a window function and taking Fourier transform of each segment. STFT has limited time-frequency resolution which is determined by the size of the window used. The uncertainty principle prohibits the usage of arbitrarily small duration and small bandwidth windows. A fundamental resolution trade-off exists: a smaller window has a higher time resolution but a lower frequency resolution, whereas a larger window has a higher frequency resolution but a lower time resolution. Hence, STFT is not capable of analyzing transient signals that contain high and low frequency components simultaneously.

B. Quadratic Time-Frequency Transforms

Cohen, in 1966, showed that all the existing bilinear time-frequency distributions could be written in a generalized time-frequency form. In addition, this general form can be used to facilitate the design of new time-frequency transforms [4]. The prominent members of Cohen's class include WVD, pseudo Wigner-Ville distribution, Choi-Williams distribution, cone-shaped distribution and adaptive kernel representation [5].The WVD was originally developed in the area of quantum mechanics by Wigner [6] and then introduced for signal analysis by Ville [7]. Compared to STFT, WVD has much better time and frequency resolution. But the main drawback of the WVD is the cross-term interference. This interference phenomenon shows frequency components that do not exist in reality and considerably affect the interpretation of the time

frequency plane. Cross-terms are oscillatory in nature and are located midway between the two components [4]. Presence of cross-terms severely limits the practical applications of WVD. Various modified versions of WVD have been developed to reduce cross-terms. These techniques include distributions from Cohen's class by Cohen [4], non-linear filtering of WVD by Arce and Hasan [8], S-method by Stankovic [9], polynomial WVD by Boashash and O'Shea [10]. The application of Fourier-Bessel transform to obtain a cross term free WVD distribution is explained in section V.

III. Fourier-Bessel Transform

The FBT decomposes a signal in to a weighted sum of an infinite number of Bessel functions of zeroth- order. Mathematically, the FBT $F(\rho)$ of a function $f(r)$ is represented as [11] :

$$F(\rho) = 2\pi \int_{0}^{\infty} f(r) J_0(2\pi\rho r) r dr \tag{1}$$

$$f(r) = 2\pi \int_{0}^{\infty} F(\rho) J_0(2\pi\rho r) \rho d\rho \tag{2}$$

where $J_0(2\pi\rho r)$ are the zeroth-order Bessel functions and ρ is transform variable.

FBT is also known as Hankel transform. As the FT over an infinite interval is related to the Fourier series over a finite interval, so the FBT over an infinite interval is related to the FB series over a finite interval. FB series expansion of a signal $x(t)$, in the interval $(0, T)$ is given as [1]:

$$x(t) = \sum_{r=1}^{M} A_r J_0(\frac{\alpha_r}{T} t), 0 < t < T \tag{3}$$

FB coefficients, A_r are computed by using following equation.

$$A_r = \frac{2 \int_{0}^{T} t x(t) J_0(\frac{\alpha_r}{T} t) dt}{T^2 [J_1(\alpha_r)]^2} \tag{4}$$

where α_r, r = 1,2,3,...M are the ascending order positive roots of $J_0(t) = 0$. Since Bessel function supports a finite bandwidth around a center frequency, the spectrum of the signal can be represented better using FB expansion. As the Bessel functions form orthogonal basis and decay over the time, non-stationary signals can be better represented using FB expansion [12]. It turns out to be a one-to-one relation between frequency content of the signal and the order of the

FB expansion, where the coefficients attain maximum amplitude [13]. As the center frequency of the signal is increased, it is observed that the order of the FB Coefficients is increased. Similarly there is a relationship between the bandwidth of the signal and the range of FB Coefficients. In particular, the range of FB Coefficients increases with the increase in the bandwidth of the signal [14]. Since both amplitude modulation (AM) and frequency modulation (FM) are part of the Bessels's basis function, the FB expansion can represent the reflected signal from a rotating target more efficiently.

IV. Fractional Fourier Transform

Fractional Fourier transform (FrFT) is the generalization of the classical Fourier transform. The applications of FrFT can be found in signal processing, communications, signal restoration, noise removal and in many other science disciplines. It is a powerful tool used for the analysis of time-varying signals. The FrFT is a linear operator that corresponds to the rotation of the signal through an angle i.e. the representation of the signal along the axis *u*, making an angle *a* with the time axis. The *a* th order Fractional Fourier Transform of the function f(*u*) is defined as [15]:

$$f_a(u) = \int f(u')K_a(u,u')du', \tag{5}$$

$$K_a(u,u') = A_\phi exp[i\pi(\cot\phi u^2) - 2\csc\phi \ uu' + \cot\phi u^2] \tag{6}$$

where

$$\phi = \frac{a\pi}{2} \tag{7}$$

$$A_\phi = \sqrt{1 - i\cot\phi} \tag{8}$$

For $a = 1$, we find that $\phi = \frac{\pi}{2}$, $A_\phi = 1$ and

$$f_1(u) = \int_{-\infty}^{\infty} \exp(-i2\phi uu')f(u')du' \tag{9}$$

for $a = 0$, FrFT reduces into identity operation. For $a = 1$, FrFT is equal to standard FT of f(*u*). For $a =$ -1, FrFT becomes an inverse FT. FrFT can transform a signal either in time or in frequency domain into a domain between time and frequency. FrFT depends on the parameter a and can be interpreted as rotation by an angle ϕ in the time-frequency plane. The FrFT of a signal can also be interpreted as a decomposition of the signal in terms of chirps [16].

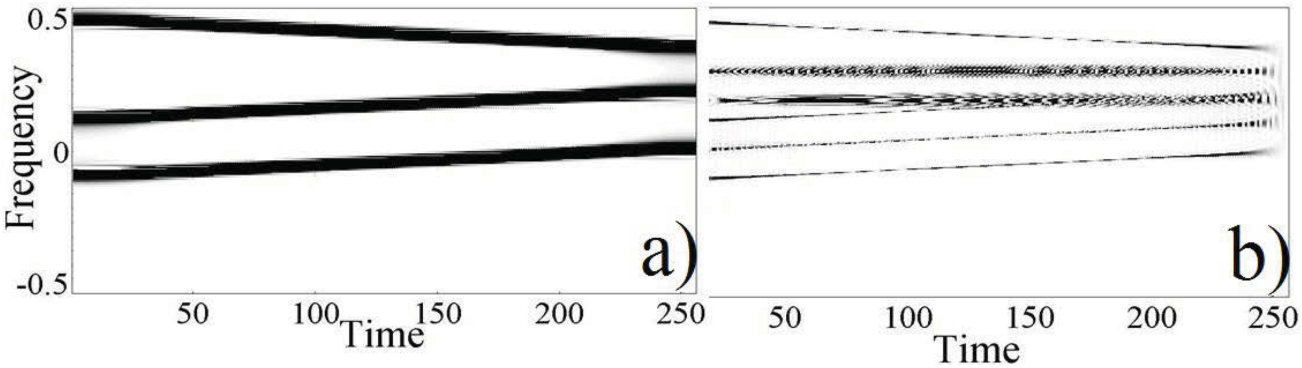

Fig. 1: a) STFT of the multi component signal, b) WVD of the multi component signal

V. Simulation Results

In this section, we demonstrate the application of the proposed method by removing the cross terms in the WVD representation of a multi-component signal. Consider a discrete time domain signal, $s[n]$, which is sum of the three linear chirps given by:

$$s[n] = \sum_{i=1}^{3} A_i \exp(2\pi \jmath f_i nT + \frac{1}{2}\beta_i (nT)^2) \tag{10}$$

where A_i are the amplitudes of the constituent signals, f_i are the fundamental frequencies, β_i are chirp rates and T is the sampling interval. Figure 1a and Figure 1b show the STFT and WVD representations of the multi component signal in the equation 10.

From Figure 1a, it is evident that STFT representation of the signal is free from cross terms but its time and frequency resolutions are poor. As expected, WVD gives good time and frequency resolution but is corrupted with the occurrence of cross terms. In order to remove these cross terms, the signal is analyzed using FBT. FB coefficients are calculated using equation (4). Figure 2 shows the FB coefficients of the multi component signal. By taking the most significant order of the FB coefficients, the multi component signal can be decomposed into its individual components. Table I shows the order of the significant FB coefficients that are selected for each chirp signal. Individual components are reconstructed by applying IFBT using the selected FB coefficients. Figure 3a, 3b and 3c show the WVD representation of each component of the multi-component signal. Figure 3d shows the plot obtained by adding WVD representations of the three linearly frequency modulated (LFM) signals together.

Results in the Figure 3 show that the occurrence of cross terms in WVD can be eliminated,

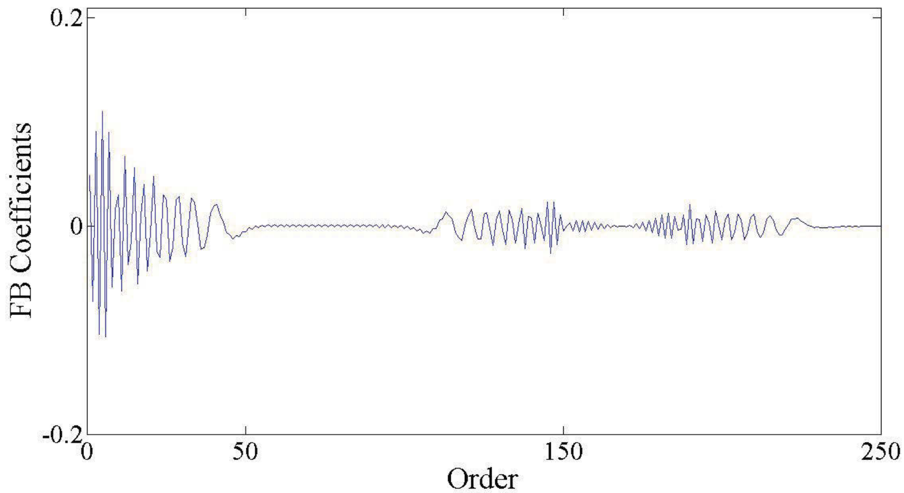

Fig. 2: FB Coefficients of the multi component signal.

signal	Required FB Coefficients
chirp 1	(1-45)
chirp 2	(108-150)
chirp 3	(151-230)

TABLE I

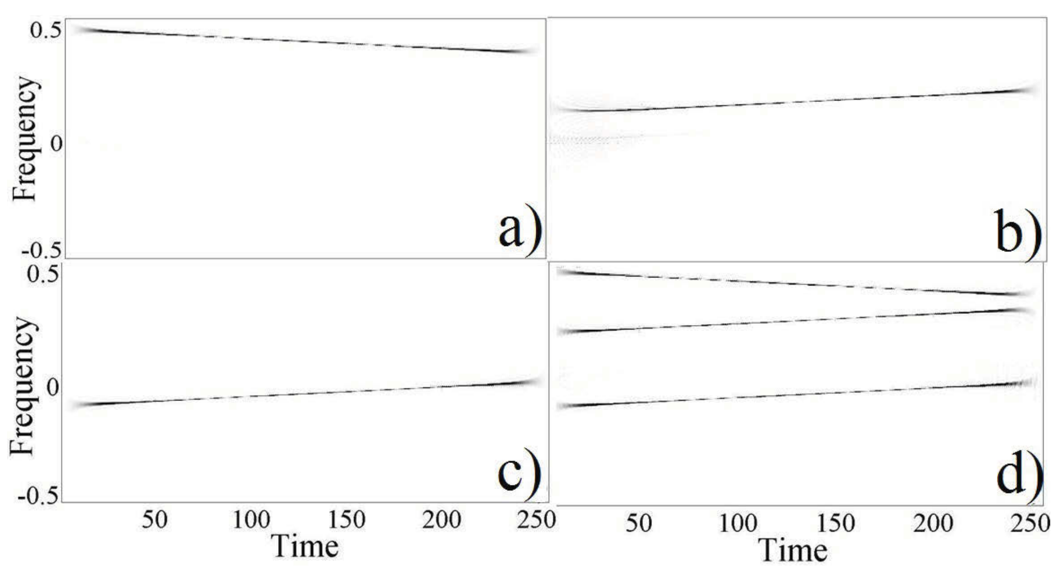

Fig. 3: $(a-c)$ are WVD of first, second and third LFM chirp respectively. d) FB-WVD plot of multi component signal.

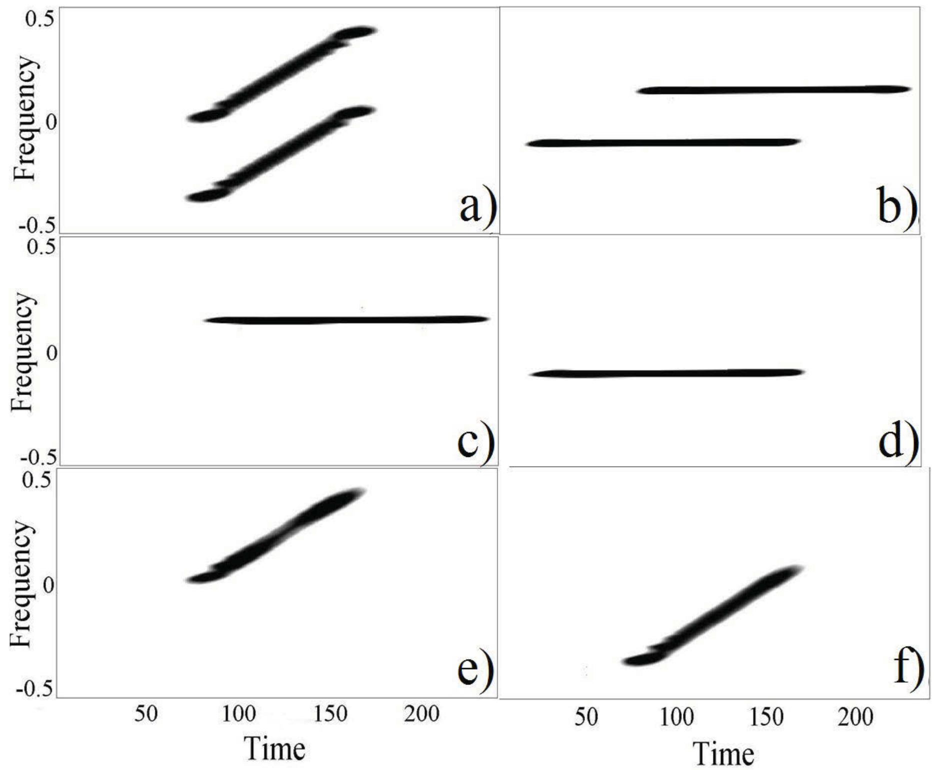

Fig. 4: Separation of two LFM components using Fractional Fourier Transform and Fourier-Bessel Transform.

if the multi component signal is decomposed into its individual components, by expanding the signal using FB series and applying WVD to the constituent signals separately. Using FBT, we can separate the components of the multi-component signals, if their frequencies do not over lap in the frequency domain. But if their frequencies overlap in time and/or frequency domain, it is not possible to separate them using FBT. By using FrFT and FBT, we can separate the components of the multi-component signal whose frequencies overlap in time and/or frequency domain.

Figure 4a shows the STFT representation of the two LFM signals whose frequencies overlap in the frequency domain. Time-frequency characteristics of the signal was rotated by 36^o in the clockwise direction by using FrFT, such that their frequency components do not overlap in the frequency domain. Figure 4b shows the STFT representation of the signal after rotation. Now using the FBT, the two frequency components of the multi-component signal were separated.

Figure 4c and 4d show the separated components of the signal. After the separation of the components, time-frequency characteristics of the signal was rotated by 36^o in the counterclockwise direction using FrFT. Figure 4e and Figure 4f show the separated LFM components. It should be emphasized here that this approach works well for any number of chirps with different angles.

VI. Experimental Results

In this section, we demonstrate the application and effectiveness of the FB-TF method, with three different types of radar data obtained in various scenarios.

A. Rotating reflectors

Experimental trials were conducted to investigate and determine the m-D radar signatures of targets using an X-band radar. The target used for this experimental trial was a spinning blade with corner reflectors attached. These corner reflectors were designed in such a way that they reflect electromagnetic radiation with a minimal loss. These controlled experiments can simulate the rotating type of objects, generally found in an indoor environment such as a rotating fan and in an outdoor environment such as a rotating antenna or rotors. Controlled experiments allow us to set the desired rotation rate of the target, to cross check and assess the results. The target employed in this experiment was at a range of 300 m from the radar. STFT representation is utilized in order to depict the m-D oscillation [17].

Figure 5a shows the STFT representation of the signal obtained from one rotating corner reflector, facing the radar. From the time-frequency signature, we can observe that the m-D of the rotating corner reflector is a time-varying frequency spectrum. Figure 5a clearly shows the sinusoidal motion of the corner reflector. The second weaker oscillation represents the reflection from the counter weight that was used to stabilize the corner reflector during the operation. It also contains a constant frequency component which is due to reflection from stationary body of the corner reflector. FBT was utilized in order to separate stationary component from the rotating component. Figure 5b shows the time-frequency signature of the extracted oscillating signal. Figure 5c shows the time-frequency signature of the extracted body signal. The rotation rate of the corner reflector is directly related to the time interval of the oscillations. The estimated rotation rate of the corner reflector was about 60 rpm. Rotation rates estimated by the time-frequency analysis agree with the actual values.

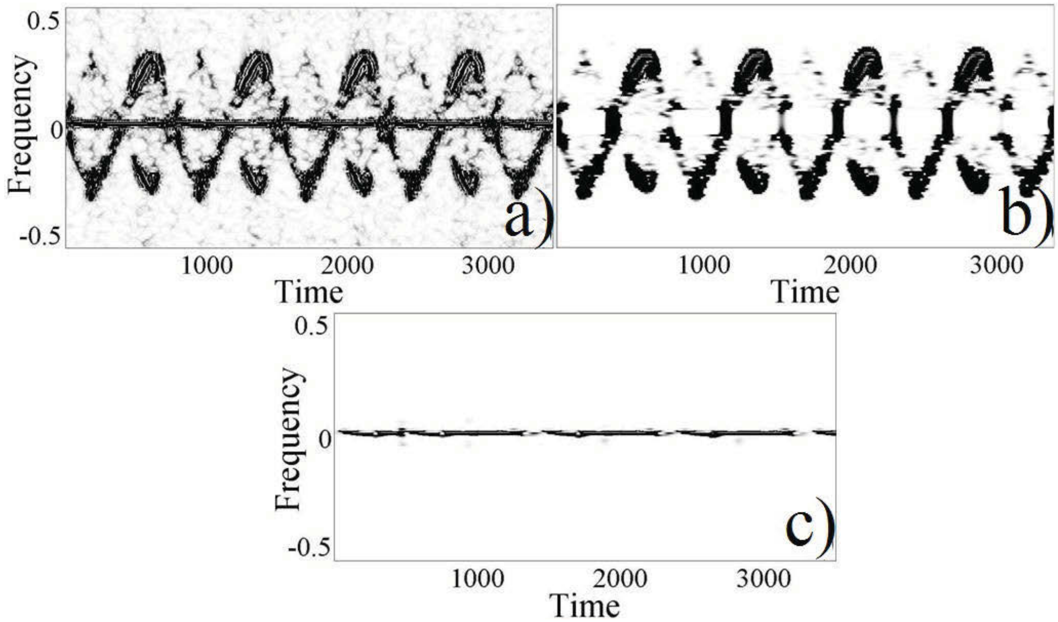

Fig. 5: a) TF signature of the signal from one rotating corner reflector facing the radar. b) TF signature of the extracted oscillating signal. c) TF signature of the extracted body signal.

B. Rotating antenna in SAR

Radar returns were collected from rotating antenna using APY-6 radar in SAR scenario. Using these data sets, we extracted the m-D features relating to a rotating antenna. The m-D features for such rotating targets may be seen as a sinusoidal phase modulation of the SAR azimuth phase history. The phase modulation may equivalently be seen as a time-varying Doppler frequency [21]. The Fourier transform of the original time series is shown in Figure 6. The rotating antenna is located close to the zero Doppler and cannot be detected using FT method. Original time series was decomposed using FBT and rotating and stationary components of the signal were captured by different order of FB coefficients. Stationary signal and oscillating signals were reconstructed by applying IFBT on the selected coefficients.

Figure 7a illustrates the time-frequency signature of the original signal and Figure 7b illustrates the time-frequency signature of the extracted oscillating signal where as Figure 7c illustrates the time-frequency signature of the extracted body signal. Using the time-frequency plot, the rotation rate of the antenna is estimated by measuring the time interval between the peaks. The period is the time interval between peaks [21]. As an example, in Figure 7b, there are 3 peaks. The

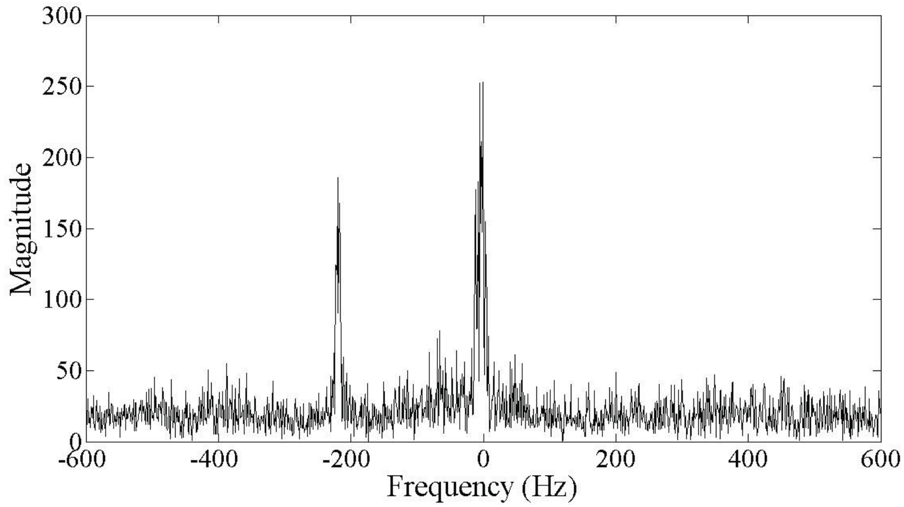

Fig. 6: The Fourier Transform of the original time series.

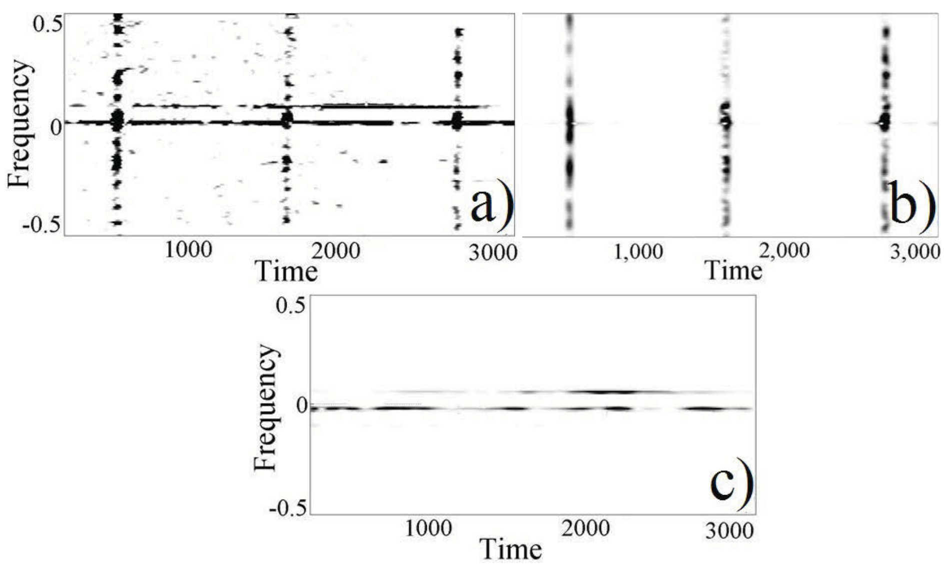

Fig. 7: a) TF signature of the original signal. b) TF signature of the extracted oscillating signal. c) TF signature of the extracted body signal.

time interval between peak 1 and 2, between 2 and 3, and between 1 and 3 were measured [18]. The average value was then used to estimate the rotation rate. The estimated rotation rate is 4.8 seconds, which is very close to the actual value of 4.7 seconds.

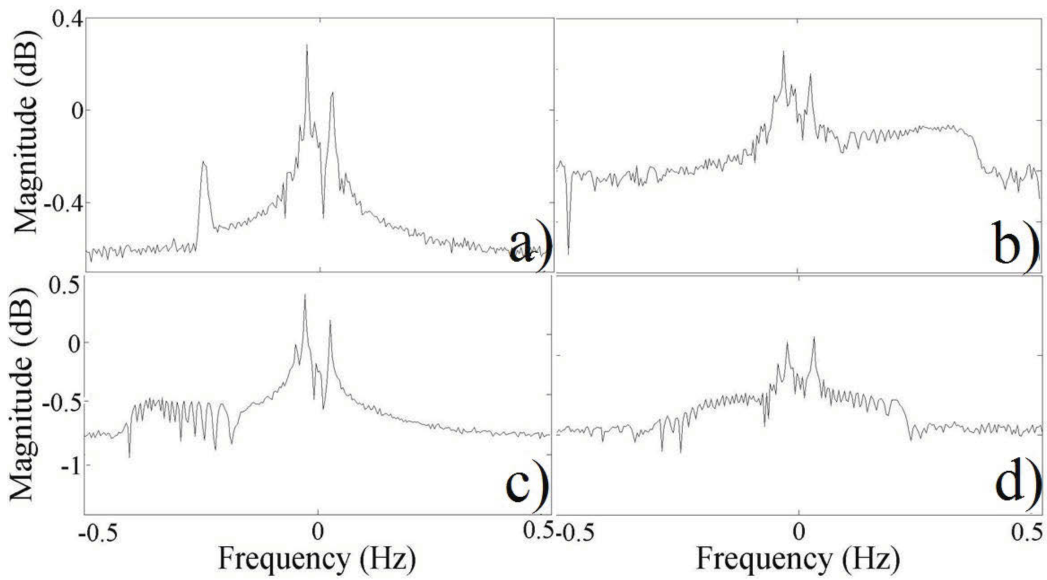

Fig. 8: a) FT of the signal 1 - Non-accelerating target far from Bragg's lines. b) FT of the signal 2 - Accelerating target far from Bragg's lines. c) FT of the signal 3 - Target very close to Bragg's lines. d) FT of the signal 4 - Target crossing the sea clutter.

C. Manoeuvring air target in sea-clutter

The signals used in the following analysis were collected from the experimental air craft. It was performing manoeuvres and being tracked by a high frequency surface wave radar (HFSWR). Since the sea clutter is more stronger than the target signal, detecting a target in the presence of the sea clutter is a challenging problem. For efficient detection and extraction of the target features, target signal has to be separated from the sea clutter and should be analyzed using time-frequency analysis [5]. One way to separate the target signal from the sea clutter is to use digital filtering techniques in Frequency domain.

1) Filtering in Frequency domain: The Fourier spectra of the four signals are shown in Figures 8a, 8b, 8c and 8d respectively. We observe that the target signal is buried in the background that consists of sea clutter and noise (thermal and atmospheric). Here the sea clutter is due to Bragg scattering from the surface of the ocean [5]. The Fourier spectra in Figure 8 contained two large spectral lines around the zero Doppler, which correspond to the sea clutter components. Figure 8c clearly illustrates that when the target is accelerating close to zero frequency or sea clutter FT method fails to provide optimum detection performance [5].

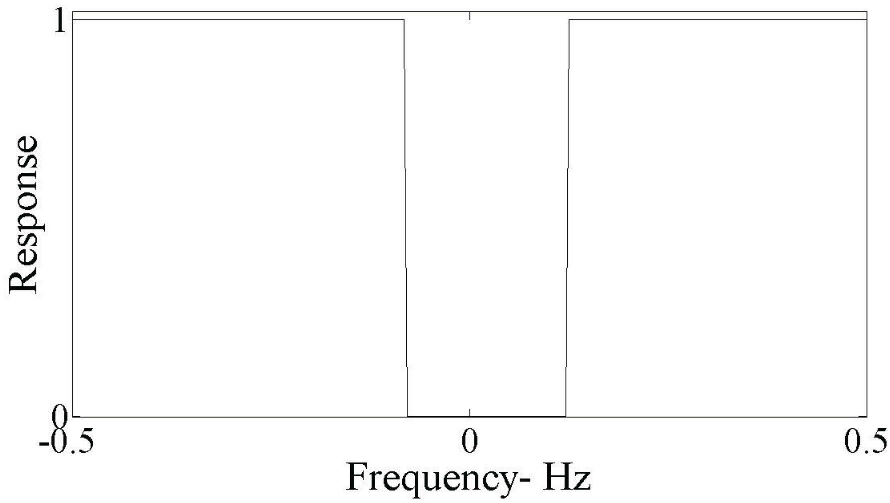

Fig. 9: Band-rejection filter.

Since the sea clutter appeared around zero Doppler, it can be removed using digital filtering techniques in frequency domain. Figure 9 shows the band-rejection filter that was used to filter the sea clutter. Figure 10a, Figure 10c and Figure 10e show the STFT plots of the three signals respectively. Figure 10b, Figure 10d and Figure 10f show the results of separating target from the sea clutter using the band-rejection filter.

Above results show that the target signal and the sea clutter can be separated using filtering techniques in frequency domain. But these filtering techniques fail to separate the target signal from the sea clutter when the target signal crosses the sea clutter. Figure 10g shows the STFT representation of the target signal crossing the sea clutter and Figure 10h shows the STFT representation of the target signal, after the sea clutter is removed using the band-rejection filter. Above results show that it is not possible to separate the target signal and sea clutter if the target is crossing the sea clutter. In the next section, we proposed a method to separate the target signal and sea clutter even when the target signal crosses the sea clutter.

2) Filtering using FB-TF method: Radar returns were analyzed using FBT and FB coefficients were calculated using equation 4. Figure 11 shows the plot of FB coefficients of the signal 1. We can observe that returns from the sea clutter were captured by the lower order FB coefficients and the target signal was captured by the higher order FB coefficients of Fourier-Bessel basis functions. Since the target signal and sea clutter are captured by different orders of

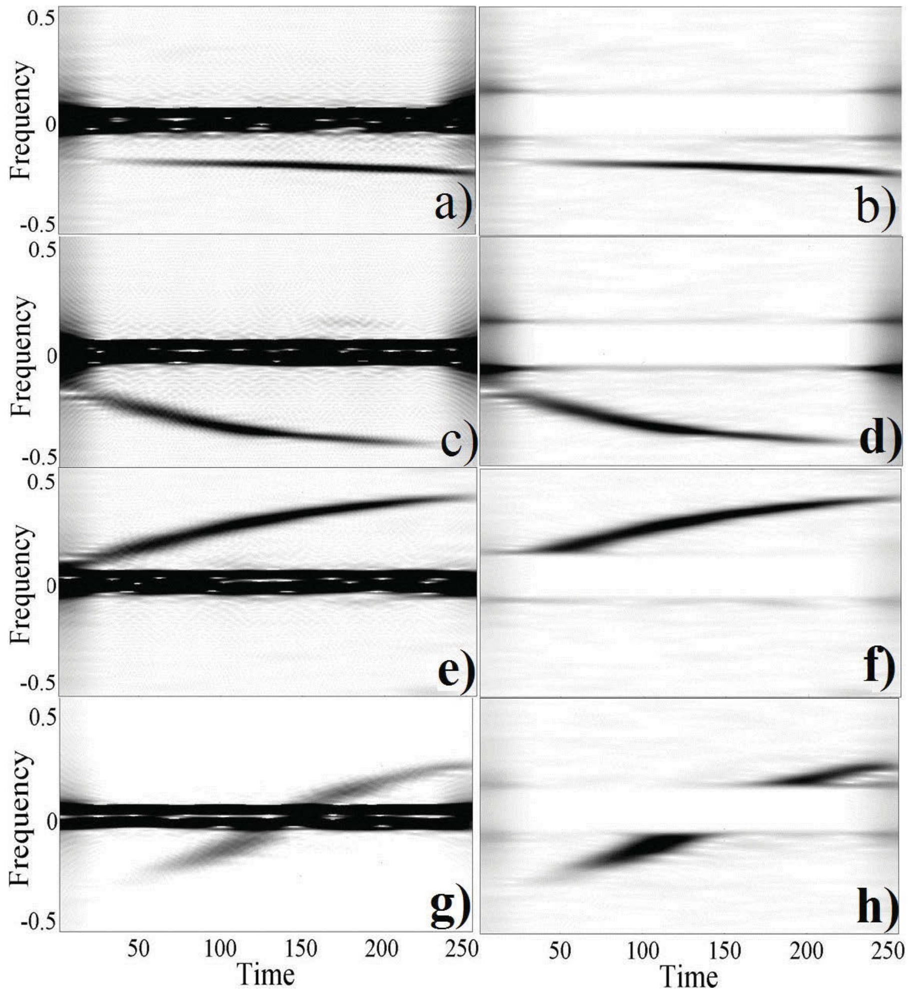

Fig. 10: a) STFT of signal 1. b) STFT of signal 1 after sea clutter is removed. c) STFT of signal 2. d) STFT of signal 2 after sea clutter is removed. e) STFT of signal 3. f) STFT of signal 3 after sea clutter is removed. g) STFT of the signal 4. h) STFT of the signal 4 after sea clutter is removed.

FB coefficients, we can easily separate the target from the sea clutter. Table II contains selected FB coefficients for sea clutter and target for three signals.

Target signal was reconstructed by applying IFBT on the selected FB coefficients of the target. After the target signal is separated from the sea clutter, time-frequency representations like STFT and WVD were used to extract more information from it. Plots in Figure 12 show the results of STFT and FB-STFT methods for signals 1, 2 and 3. By using FBT, we can separate the target signal from the sea clutter more efficiently even when the target signal is very close to sea

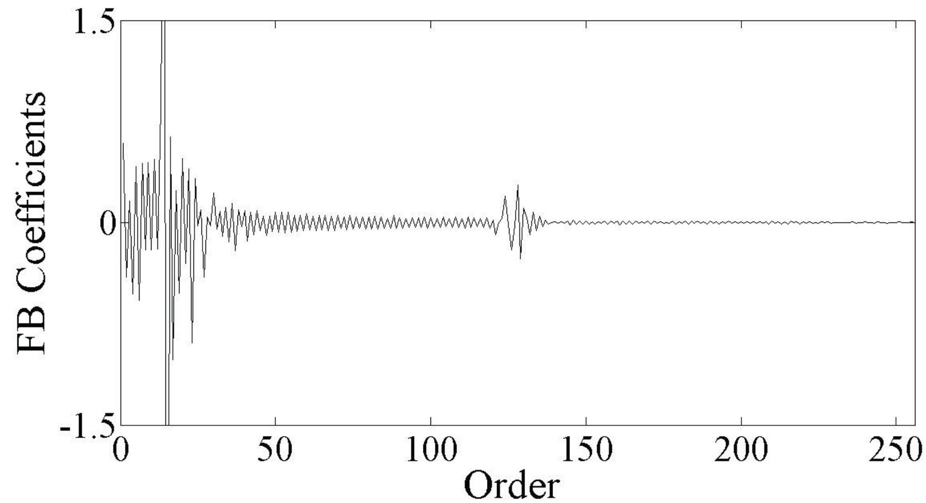

Fig. 11: FB coefficients of the signal 1

Signals	Sea Clutter Coefficients	Target Coefficients
Signal 1	(1-25)	(120-138)
Signal 2	(1-25)	(44-84)
Signal 3	(1-25)	(141-199)

TABLE II

clutter. In the case of target signal crossing the sea clutter, as shown in Figure 13, it is possible to separate them using FrFT and FBT. By using FrFT, time-frequency signature of the signal is rotated in counter clockwise direction through an angle θ such that, the target signal is aligned perpendicular to the frequency axis at around zero Doppler. Now the signal is analyzed using FBT and the target signal is separated by selecting the higher order FB coefficients corresponding to the target signal. Time-frequency (TF) signature of the target signal is reconstructed by applying IFBT on the selected FB coefficients. Now the TF signature of the target signal is rotated in the clockwise direction through an angle θ to obtain the separated target signal. Figure 13b and Figure 13c show the FB-STFT and FB-WVD representations of the signal after the target is separated from sea clutter.

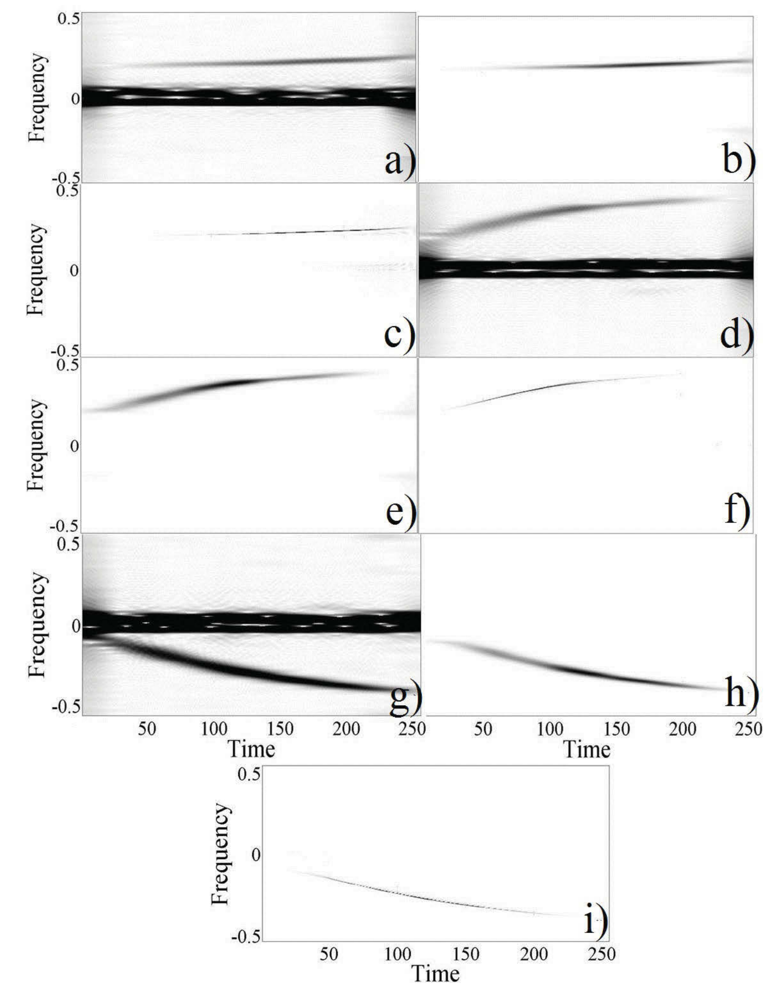

Fig. 12: Figures (a,d,g), (b,e,h) and (c,f,i) show the results of STFT, FB-STFT and FB-WVD analysis for three signals, respectively.

VII. Conclusion

By applying the proposed method to simulated and several experimental data sets, the effectiveness of this FB-TF technique is confirmed. This method combines both FBT and time-frequency analysis to extract the m-D features of the radar returns. By applying the proposed method to the rotating antenna data and to the rotating corner reflectors data, the potential of the proposed method is ascertained. From the extracted m-D signatures, the information about the target's micro-motion dynamics such as rotation rate is obtained. The experimental results agree with the expected outcome. FB-TF proves to be a useful tool in the reduction of the sea clutter and the target enhancement. Using FB-TF method, we could separate the target from the strong sea

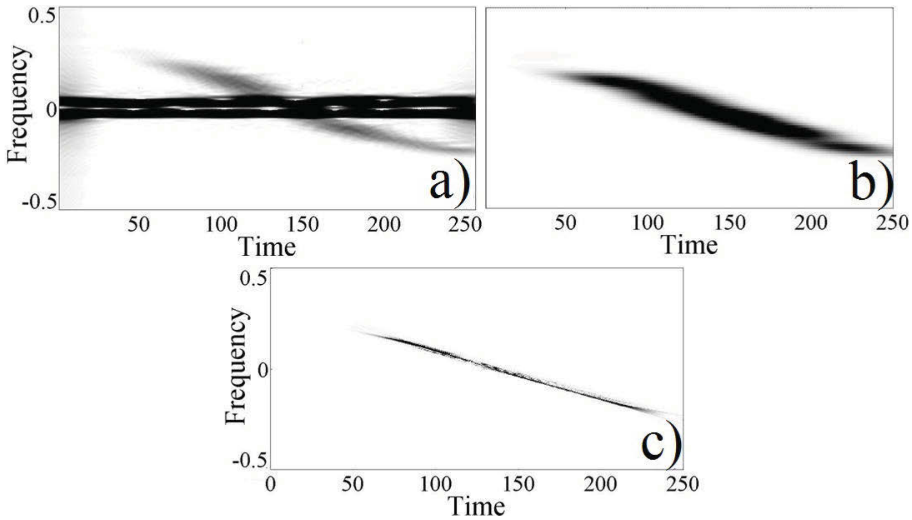

Fig. 13: a) STFT representation of the original signal. b) FB-STFT representation of the target. c) FB-WVD representation of the target.

clutter. In the case of target signal crossing the sea clutter, target signal was separated from the sea clutter using the FrFT and FBT. Results demonstrate that the proposed method could be used as a potential tool for detecting and enhancing low observable maneuvering, accelerating air targets in the littoral environments.

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